Department of Mathematics

École Centrale School of Engineering, Mahindra University, Hyderabad

Syllabus for Written Test for Ph.D. Admission Fall - 2024

Linear Algebra:

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Complex Analysis:

Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, and Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Ordinary Differential Equations (ODEs):

Existence and Uniqueness of solutions of initial value problems for first-order ordinary differential equations, singular solutions of first-order ODEs, and system of first-order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm- Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs):

Lagrange and Charpit methods for solving first-order PDEs, Cauchy problem for first order PDEs. Classification of second-order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis:

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Abstract Algebra:

Definition of Groups, Subgroups and Factor Groups, Lagrange's Theorem, Homomorphisms, Normal Subgroups. Quotients of Groups. Basic Examples of Groups including Symmetric Groups, and matrix Groups, Sylow's Theorems, Finite Fields.

Real Analysis:

Limit of Functions. Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite Limits and Limit at Infinity. Derivative of a Real Function. Mean Value Theorem, Continuity of Derivatives, L'Hospital's Rule, Derivatives of Higher Order, Taylor's Theorem.

Statistics and Probability Theory:

Probability, conditional probability, independent events, total probability, and Baye's theorem. Random Variable, Probability density function, distribution function, mathematical expectation, variance, Discrete Distributions – Binomial, Poisson, Continuous Distribution – Normal distribution. Central Limit Theorem, Joint Probability Distribution, Transformations of Random Variables, Confidence Intervals, Maximum Likelihood Estimators, Hypothesis Testing.

Functional Analysis: Metric spaces, Normed linear spaces and Banach spaces, Bounded linear operators/functionals, Hilbert spaces.

Topology:

Basic Concepts of Topology, Bases, Subspace Topology, Product Topology, Metric Topology, Connectedness, Compactness, Countability and Separation Axioms, Urysohn's Lemma.